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Nonlinear diffusion equation using Lattice-Boltzmann

F. Fonseca(1)

(1) Departamento de Física, Universidad Nacional de Colombia, Bogotá-Ciudad Universitaria, e-mail: frfonsecaf@unal.edu.co.

Abstract – We apply the lattice Boltzmann technique to solve the nonlinear diffusion equation.

Many physical systems are described by nonlinear diffusion processes. We can find a wide range of problems in physical, chemical, biological, statistical, and engineering sciences.

In physics we have e.g., the problem of the ascent of warm moist air in atmosphere, which has been modeled using a modification of K.P.Z. equation, taking away the noise term, and the remaining equation becomes a non-linear diffusion type equation [1]. Also the uphill diffusion process, which consist in diffusion up a concentration gradient which describes several phenomena as diffusion in ceramic, metallic systems and silicate melts [2]. Further, nonlinear diffusion is applied to the study chemical reactions heat and mass transfer [3], theory of combustion, processes in biology and ecology.

On the other hand, lattice Boltzmann has been applied with success to many problems in Physics. The method is suitable to simulate hydrodynamics systems [4], magneto-hydrodynamics systems [5], multiphase fluids [6], charge distribution in electrolytes [7], chemical-reactive flows and flow to porous media [8]. The method comes from the pioneer work of Wolfram on lattice-gas automata [9]. Lattice Boltzmann collects the physics in a bottom-up approach. Basically, the space is divided into a regular lattice and on the site is assigned a velocity distribution that comes from the discretization of the Boltzmann equation, which describes the statistical distribution of a system of particles in a fluid. At each time step, particles jump or move to the nearest lattice site along the direction of motion, where they interact with others particles that arrive to the same lattice site. The result of the collision is determined by the discretized Boltzmann equation for the new particle distribution at the site. At last, the distribution is updated.

Based on the lattice-Boltzmann technique we find de solution to the next nonlinear diffusion equation [10].

$$\frac{\partial \phi}{\partial t} = \frac{\partial}{\partial x}\left[\phi^{-m}\frac{\partial \phi}{\partial x}\right]$$

Figure 1 shows the simulation result:

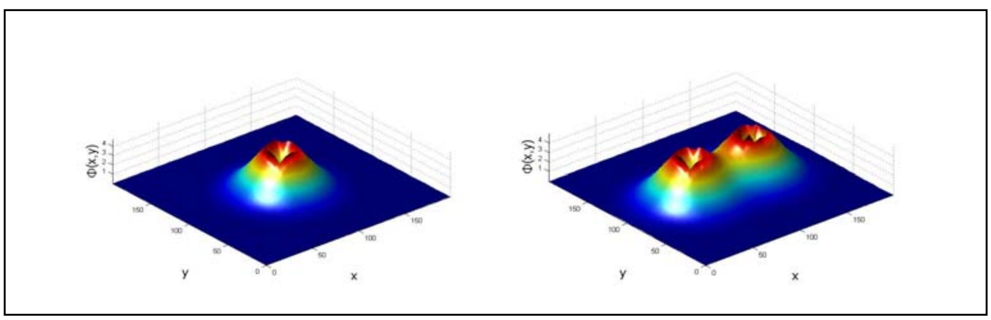

Figure 1: 2d solution of the nonlinear diffusion equation with one and two sources.

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