

## Method of convergence used for calculating electron transport described by 3D invariant imbedding differential equations.

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The electron transport processes in spectroscopy techniques described by the invariant imbedding theory is here revisited. We report a convergence method to obtain analytical solutions of the 3D integro-differential equations. The method was used successfully in calculating the atomic number and energy dependence of the backscattered electrons fraction (Fig.1). Also the fraction of absorbed electrons was calculated as function of incident angles (Fig.2). Using a states ladder model for the electron's energies, the method provides a tool for testing physical parameters involved in the transport theory, like elastic and inelastic cross sections.

Considering structural and chemical characterization methods, the electron and photon transport has several applications including electron probe microanalysis (EPMA), X-ray fluorescence analysis, electron-beam-induced-current, Auger Electron Spectroscopy, Electron Energy Loss Spectroscopy, X-ray Photoelectron Spectroscopy (XPS). The mentioned characterization techniques are based on the strong interaction of electrons and or photons with the matter and in the signals that they produce. Boltzmann transport equation and Monte Carlo calculation are the used methods for interpretation of the signals in spectroscopic techniques and characterization. However, have same disadvantage, as they are uneconomical methods since much information about internal fluxes is obtained that is quite useless to the experimenter.

The invariant imbedding method is a mathematic technique based directly upon the physical processes. The traditional second order Boltzmann equations, subject to boundary conditions, are transformed into nonlinear functional equations subject only to initial conditions in space and time coordinates [1-3]. Considering the convergence of an approximate solution of the differential equations to the exact solution in a medium of infinite size, the distributions of the backscattered and the absorbed electrons fraction are obtained.

The theoretical result leads to reasonably accurate results for the atomic number dependence (Figure 1a). However, in the case of energy spectrum, our theoretical model predicts a maximum at the incident energy and there is not backscattered electrons above this energy. Actually, in experimental results the maximum in the spectrum appear below the incident energy. The discrepancy could be explained because our model consider the possibility of elastic collision and actually all collision occur with certain amount of energy loses.

A qualitative behavior of the theoretical results for the fraction of absorbed electrons as a function of incident angle is presented in figure 2. In agree with experimental results the maximum occur at normal incidence of the electron beam.

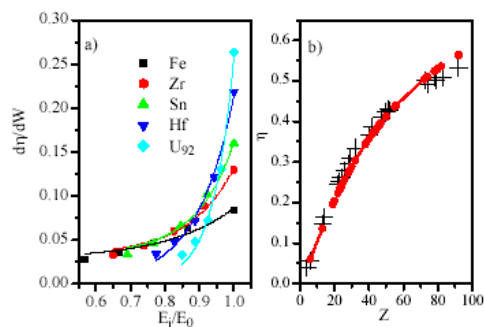


Figure1: a) Energy spectrum of backscattered electrons from samples of different atomic number Z. b) backscattered electrons fraction as a function of atomic number. Theoretical values, red points.

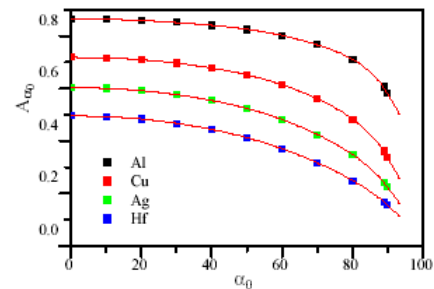


Figure 2: The fraction of absorbed electrons as a function of incident angle.

### References:

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